Cumhuriyet Science Journal

e-ISSN: 2587-246X
ISSN: 2587-2680

Cumhuriyet Sci. J., 42(1) (2021) 88-98
http://dx.doi.org/10.17776/csj.689759

$(G'/G, 1/G)$-expansion method for analytical solutions of Jimbo-Miwa equation

Asıf YOKUŞ [1], ***Hülya DURUR*** [2,*]

[1]*Fırat University, Faculty of Science, Department of Actuary, Elazığ/TURKEY*

[2]*Ardahan University, Faculty of Engineering, Department of Computer Engineering, Ardahan/TURKEY*

Abstract

The main goal of this study is obtaining analytical solutions for (3+1)-dimensional Jimbo-Miwa Equation which the second equation in the well-known KP hierarchy of integrable systems. For the (3+1DJM) equation, hyperbolic, trigonometric, complex trigonometric and rational traveling wave solutions have been constructed by applying the $(G'/G, 1/G)$-expansion method. Then, real and imaginary graphics are presented by giving special values to the constants in the solutions obtained. These graphics are a special solution of the (3+1DJM) equation and represent a stationary wave of the equation. The $(G'/G, 1/G)$-expansion method is an effective and powerful method for solving nonlinear evolution equations (NLEEs). Ready computer package program is used to obtain the solutions and graphics presented in this study.

Article info

History:
Received:15.02.2020
Accepted:23.12.2020

Keywords:
(3+1)-dimensional Jimbo-Miwa equation,
Exact solution,
$(G'/G, 1/G)$-expansion method,
Traveling wave solution.

1. Introduction

Non-linear science phenomena take an important place in applied mathematics and mathematical physics. The appearance of the solitary wave in nature is quite common particularly in plasmas, condensed matter physics, optical fibers, solid-state physics, chemical kinematics, electrical circuits, etc. Solutions of nonlinear evolutionary equations are the cornerstone of many physical phenomena. For example; such as signaling, ocean waves, shallow water waves, heat dissipation. In wave theory, these physical events are mathematized with traveling wave solutions. Therefore, the traveling wave solution is an indispensable instrument of both mathematics and physics. Last year, NLEEs are applied in many areas of science. Various methods have been used to obtain solutions for different types of NLEEs. Some of these are the extended trial equation method [1], the new extended direct algebraic method [2], (G'/G)-expansion method [3], $(G'/G, 1/G)$-expansion method [4], Clarkson–Kruskal (CK) direct method [5], improved tan $\phi(\xi)/2$-expansion method [6], Sumudu transform method [7], new expansion method [8], extended sinh-Gordon equation expansion method [9], the modified Kudryashov method [10], $(1/G')$-expansion method [11-15], collocation method [16,17], first integral method [18], F-expansion method [19], the modified exponential function method [20], Difference scheme method [21] and so on [22-29].

In this article, the authors attained the exact solutions of the (3+1DJM) equation. Consider the form of the (3+1DJM) equation [30],

$$u_{xxxy} + 3u_y u_{xx} + 3u_x u_{xy} + 2u_{yt} - 3u_{xz} = 0. \quad (1)$$

Jimbo-Miwa equation was examined first by Jimbo and Miwa [31] and later by several authors. Some of these are as follows: Wazwaz has been attained multiple soliton solutions of two extended (3+1DJM) equation using the simplified Hirota's method [32], Zhang has been obtained different type solutions of (3+1DJM) equation using Hirota bilinear form [33], Ma and Lee have been attained (3+1DJM) equation using Bäcklund transformation [34], Sun and Chen have been attained lump solutions, the lump–kink of (3+1DJM) equation via bilinear forms [35], Yang and Ma have been obtained Lump-type solutions of the (3+1DJM) equation by applying Hirota bilinear form [36], Liu and Jiang have been attained new exact solutions of the (3+1DJM) equation with the help of extended homogeneous balance method [37], Öziş and Aslan have been attained analytical and explicit generalized solitary solutions of the (3+1DJM) equation using the Exp-function method [38], Tang

*Corresponding author. *email address: hulyadurur@ardahan.edu.tr*
 ©2021 Faculty of Science, Sivas Cumhuriyet University

and Liang have been obtained variable separation solutions of the (3+1DJM) equation [39], Ma has been attained four classes of lump-type solutions for the (3+1DJM) equation using Hirota bilinear form [40].

2. $(G'/G, 1/G)$-Expansion Method

Consider the general form of NLEEs containing two or more arguments to be analyzed is written as follows [23]:

$$T\left(u, \frac{\partial u}{\partial t}, \frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial^2 u}{\partial x^2}, \dots\right) = 0. \quad (2)$$

Let $u = u(x, y, z, t) = U(\xi) = U,\ \xi = \alpha x + \beta y + \gamma z - ct$ and transmutation Eq. (1) can be converted into following nODE for $U(\xi)$:

$$\mathrm{L}(U, U', U'', UU', \dots) = 0, \quad (3)$$

where prime refers to derivatives related to ξ. Complexity can be reduced by integrating Eq. (3). By the nature of this method, $G(\xi) = G$ is solution function of the second-order ODE as

$$G''(\xi) + \lambda G(\xi) = \mu. \quad (4)$$

It as $\phi = \phi(\xi) = G'/G$ and $\psi = \psi(\xi) = 1/G$ supply operational esthetic. We may write derivatives of the functions described

$$\phi' = -\phi^2 + \mu\psi - \lambda, \psi' = -\phi\psi, \quad (5)$$

We can offer the behavior of solution function Eq. (4) according to the state of λ taking into account the equations given by the Eq. (5).

i) If $\lambda < 0$,

$$G(\xi) = c_1 \sinh\left(\sqrt{-\lambda}\xi + c_2 cosh\sqrt{-\lambda}\xi\right) + \frac{\mu}{\lambda}, \quad (6)$$

whereas c_1 and c_2 are real numbers. Considering Eq. (6);

$$\psi^2 = \frac{-\lambda}{\lambda^2\sigma + \mu^2}\left(\phi^2 - 2\mu\psi + \lambda\right), \sigma = c_1^2 - c_2^2, \quad (7)$$

Eq. (7) is obtained.

ii) If $\lambda > 0$,

$$G(\xi) = c_1 \sin\left(\sqrt{\lambda}\xi + c_2 cos\sqrt{\lambda}\xi\right) + \frac{\mu}{\lambda}, \quad (8)$$

whereas c_1 and c_2 are real numbers. Considering Eq. (8),

$$\psi^2 = \frac{\lambda}{\lambda^2\sigma - \mu^2}(\phi^2 - 2\mu\psi + \lambda), \sigma = c_1^2 + c_2^2, \quad (9)$$

Eq. (9) is written.

iii) If $\lambda = 0$,

$$G(\xi) = \frac{\mu}{2}\xi^2 + c_1\xi + c_2, \quad (10)$$

here c_1 and c_2 are real numbers. Considering Eq. (10),

$$\psi^2 = \frac{1}{c_1^2 - 2\mu c_2}(\phi^2 - 2\mu\psi). \quad (11)$$

Eq. (11) is written.

For ψ and ϕ polynomials, solution of Eq. (3) is

$$u(\xi) = \sum_{i=0}^{n} a_i \phi^i + \sum_{i=1}^{n} b_i \phi^{i-1}\psi. \quad (12)$$

Here, a_i and b_i are real numbers be identified. n is a positive equilibrium term which may be attained by comparing the maximum order derivative with the maximum order nonlinear term in Eq. (8). Whether Eq. (12)

is written in Eq. (3) along with equations (5), (7), (9), or (11) is written. Each coefficient of $\phi^i\psi^i$ $(i = 0,1,\dots,n)(j = 1,\dots,n)$ terms of the attained polynomial functions are equals zero and there is a system of algebraic equations for $a_i,\ b_i,\ c_1,\ c_2,\ \alpha,\ \beta,\ c$, and $\lambda,\ (i = 0,1,\dots,n)$. The necessary coefficients are obtained by solving an algebraic equation with the help of computer package programs. Obtained coefficients are put into Eq. (12) and $U(\xi)$ solution function of the ODE given as (3) is attained and if $\xi = \alpha x + \beta y + \gamma z - ct$ transmutation is employed, we will attain exact solution $u(x,y,z,t)$ of Eq. (2).

3. Solutions of the (3+1DJM) Equation

We consider Eq. (1) and using transformation $u = u(x,y,z,t) = U(\xi) = U,\ \xi = \alpha x + \beta y + \gamma z - ct,$

where β, γ, α and c are constants, once obtained (ODE) after integration, we get

$$\alpha^3\beta U''' + 3\beta\alpha^2(U')^2 - (2\beta c + 3\alpha\gamma)U' = 0, \tag{13}$$

where the integration constant is zero. In Eq. (13), we get term $n = 1$ from the definition of balancing term and the following situation is obtained in Eq. (5),

$$U(\xi) = a_0 + a_1\phi[\xi] + b_1\psi[\xi]. \tag{14}$$

Replacing Eq. (14) into Eq. (13) and the coefficients of Eq. (1) are equal to zero, we may establish the following algebraic equation systems

$$Const: 2c\beta\lambda a_1 + 3\alpha\gamma\lambda a_1 - 2\alpha^3\beta\lambda^2 a_1 + \frac{3\alpha^3\beta\lambda^2\mu^2 a_1}{\mu^2+\lambda^2\sigma} + 3\alpha^2\beta\lambda^2 a_1^2 - \frac{3\alpha^2\beta\lambda^2\mu^2 a_1^2}{\mu^2+\lambda^2\sigma} = 0,$$

$$\phi[\xi]: -\frac{6\alpha^3\beta\lambda^2\mu b_1}{\mu^2+\lambda^2\sigma} + \frac{6\alpha^2\beta\lambda^2\mu a_1 b_1}{\mu^2+\lambda^2\sigma} = 0,$$

$$\phi[\xi]^2: 2c\beta a_1 + 3\alpha\gamma a_1 - 8\alpha^3\beta\lambda a_1 + \frac{3\alpha^3\beta\lambda\mu^2 a_1}{\mu^2+\lambda^2\sigma} + 6\alpha^2\beta\lambda a_1^2 - \frac{3\alpha^2\beta\lambda\mu^2 a_1^2}{\mu^2+\lambda^2\sigma} - \frac{3\alpha^2\beta\lambda^2 b_1^2}{\mu^2+\lambda^2\sigma} = 0,$$

$$\phi[\xi]^3: -\frac{6\alpha^3\beta\lambda\mu b_1}{\mu^2+\lambda^2\sigma} + \frac{6\alpha^2\beta\lambda\mu a_1 b_1}{\mu^2+\lambda^2\sigma} = 0, \tag{15}$$

$$\psi[\xi]: -2c\beta\mu a_1 - 3\alpha\gamma\mu a_1 + 5\alpha^3\beta\lambda\mu a_1 - \frac{6\alpha^3\beta\lambda\mu^3 a_1}{\mu^2+\lambda^2\sigma} - 6\alpha^2\beta\lambda\mu a_1^2 + \frac{6\alpha^2\beta\lambda\mu^3 a_1^2}{\mu^2+\lambda^2\sigma} = 0,$$

$$\phi[\xi]\psi[\xi]: 2c\beta b_1 + 3\alpha\gamma b_1 - 5\alpha^3\beta\lambda b_1 + \frac{12\alpha^3\beta\lambda\mu^2 b_1}{\mu^2+\lambda^2\sigma} + 6\alpha^2\beta\lambda a_1 b_1 - \frac{12\alpha^2\beta\lambda\mu^2 a_1 b_1}{\mu^2+\lambda^2\sigma} = 0,$$

$$\phi[\xi]^2\psi[\xi]: 12\alpha^3\beta\mu a_1 - 6\alpha^2\beta\mu a_1^2 + \frac{6\alpha^2\beta\lambda\mu b_1^2}{\mu^2+\lambda^2\sigma} = 0,$$

the aim with ready computer package program, reaching the solutions of system (15) and we obtained the following stations.

If $\lambda < 0$,

Case 1.

$$a_1 = \alpha,\ b_1 = i\alpha\sqrt{\lambda}\sqrt{\sigma},\ c = \frac{-3\alpha\gamma-\alpha^3\beta\lambda}{2\beta},\ \mu = 0,\ \xi = \alpha x + \beta y + \gamma z - ct\ , \tag{16}$$

replacing values Eq. (16) into Eq. (14) and attain the following hyperbolic exact solutions for Eq. (1):

$$u_1(x,y,z,t) = \frac{2\alpha(\,c_2\sqrt{-\lambda}\mathrm{Cosh}[\sqrt{-\lambda}\xi] + c_1\sqrt{-\lambda}\mathrm{Sinh}[\sqrt{-\lambda}\xi])}{c_1\mathrm{Cosh}[\sqrt{-\lambda}\xi] + c_2\mathrm{Sinh}[\sqrt{-\lambda}\xi]} + a_0. \tag{17}$$

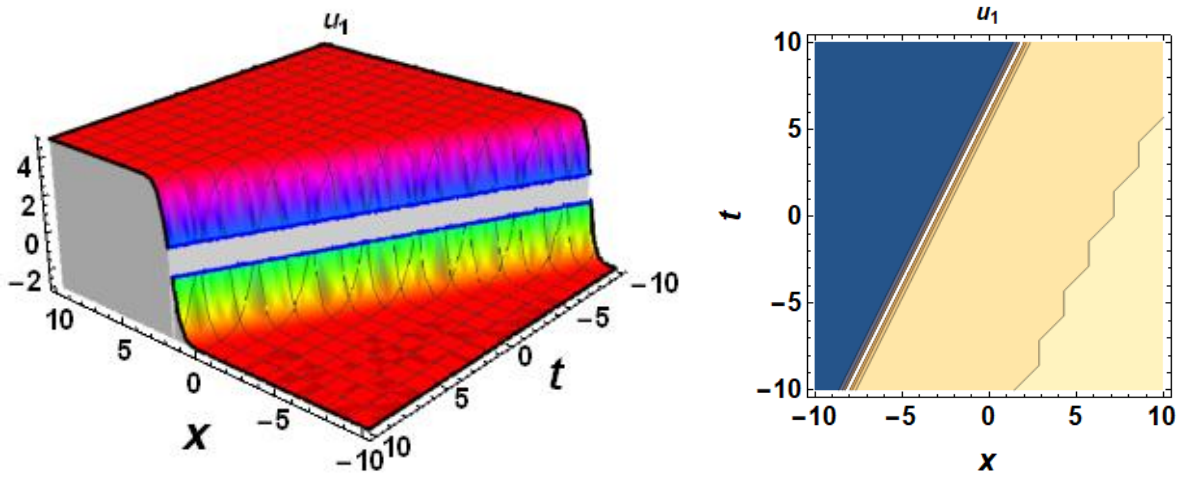


Figure 1. Graphs of $u_1(x,y,z,t)=u_1$ for $\lambda=-4,\ \beta=1,\ \gamma=5,\ \delta_1=4,\delta_2=-1,\ a_0=1,\alpha=2,\ \ c_1=1,\ \ c_2=2,$ $y=1,\ z=1.$

Case 2.

$$a_1=2\alpha,\ b_1=0,\ c=\frac{-3\alpha\gamma-4\alpha^3\beta\lambda}{2\beta},\ \mu=0,\ \xi=\alpha x+\beta y+\gamma z-ct, \tag{18}$$

replacing values Eq. (18) into Eq. (14) and attain the following hyperbolic exact solution for Eq. (1):

$$u_2(x,y,z,t)=\frac{2\alpha(c_2\sqrt{-\lambda}\mathrm{Cosh}[\sqrt{-\lambda}\xi]+c_1\sqrt{-\lambda}\mathrm{Sinh}[\sqrt{-\lambda}\xi])}{c_1\mathrm{Cosh}[\sqrt{-\lambda}\xi]+c_2\mathrm{Sinh}[\sqrt{-\lambda}\xi]}+a_0. \tag{19}$$

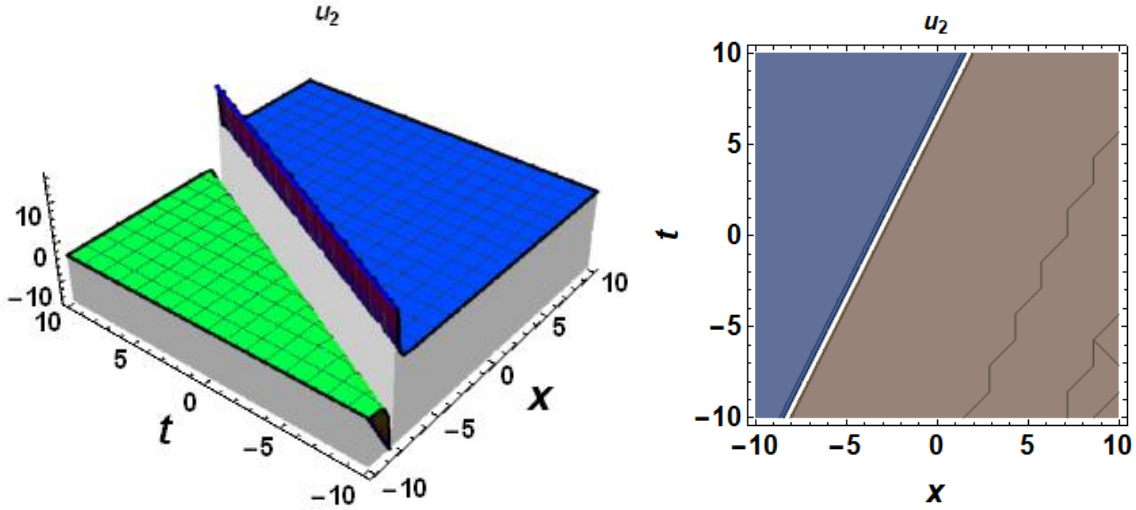


Figure 2. Graphs of $u_2(x,y,z,t)=u_2$ for $\lambda=-1,\ \beta=1,\ a_0=1,\ \gamma=5,\ \alpha=2,\ c_1=1,\ c_2=2,\ \mathrm{y}=1,\mathrm{z}=1.$

ii) If $\lambda>0,$

Case 3.

$$a_1=2\alpha, b_1=0, c=\frac{-3\alpha\gamma-4\alpha^3\beta\lambda}{2\beta}, \mu=0,\ \xi=\alpha x+\beta y+\gamma z-ct, \tag{20}$$

replacing values Eq. (20) into Eq. (14) and attain the following trigonometric exact solution for Eq. (1):

$$u_3(x,y,z,t) = \frac{2\alpha(\, c_2\sqrt{\lambda}\mathrm{Cos}[\sqrt{\lambda}\xi] - c_1\sqrt{\lambda}\mathrm{Sin}[\sqrt{\lambda}\xi])}{c_1\mathrm{Cos}[\sqrt{\lambda}\xi] + c_2\mathrm{Sin}[\sqrt{\lambda}\xi]} + a_0. \tag{21}$$

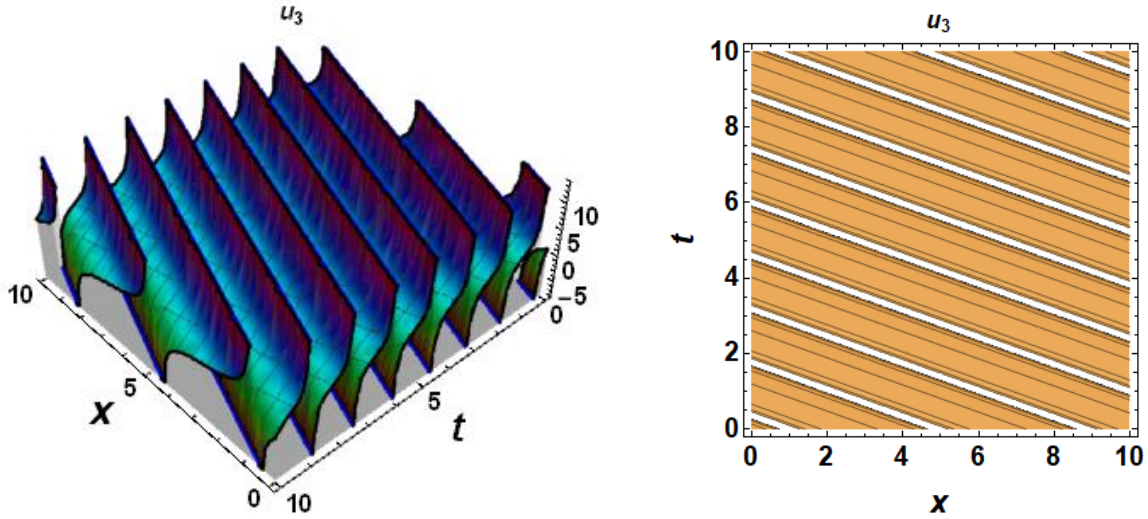


Figure 3. Graphs of $u_3(x,y,z,t) = u_3$ for $\lambda = 1,\ \beta = 1,\ a_0 = 3,\ \gamma = 1,\ \alpha = 0.8,\ \ c_1 = -1,\ c_2 = 1,\ \mathrm{y} = 0, \mathrm{z} = 0.$

Case 4.

$$a_1 = \alpha, b_1 = -\frac{\alpha\sqrt{-\mu^2+\lambda^2\sigma}}{\sqrt{\lambda}}, \beta = -\frac{3\alpha\gamma}{2c+\alpha^3\lambda},\ \xi = \alpha x + \beta y + \gamma z - ct, \tag{22}$$

replacing values Eq. (22) into Eq. (14) and attain the following trigonometric exact solution for Eq. (1):

$$u_4(x,y,z,t) = -\frac{\alpha\sqrt{(\,{c_1}^2 + {c_2}^2)\lambda^2 - \mu^2}}{\sqrt{\lambda}(\frac{\mu}{\lambda} + c_1\mathrm{Cos}[\sqrt{\lambda}(-ct+x\alpha+z\gamma-\frac{3y\alpha\gamma}{2c+\alpha^3\lambda})] + c_2\mathrm{Sin}[\sqrt{\lambda}(-ct+x\alpha+z\gamma-\frac{3y\alpha\gamma}{2c+\alpha^3\lambda})])} +$$
$$\frac{\alpha(\, c_2\sqrt{\lambda}\mathrm{Cos}[\sqrt{\lambda}(-ct+x\alpha+z\gamma-\frac{3y\alpha\gamma}{2c+\alpha^3\lambda})] - c_1\sqrt{\lambda}\mathrm{Sin}[\sqrt{\lambda}(-ct+x\alpha+z\gamma-\frac{3y\alpha\gamma}{2c+\alpha^3\lambda})])}{\frac{\mu}{\lambda} + c_1\mathrm{Cos}[\sqrt{\lambda}(-ct+x\alpha+z\gamma-\frac{3y\alpha\gamma}{2c+\alpha^3\lambda})] + c_2\mathrm{Sin}[\sqrt{\lambda}(-ct+x\alpha+z\gamma-\frac{3y\alpha\gamma}{2c+\alpha^3\lambda})]} + a_0. \tag{23}$$

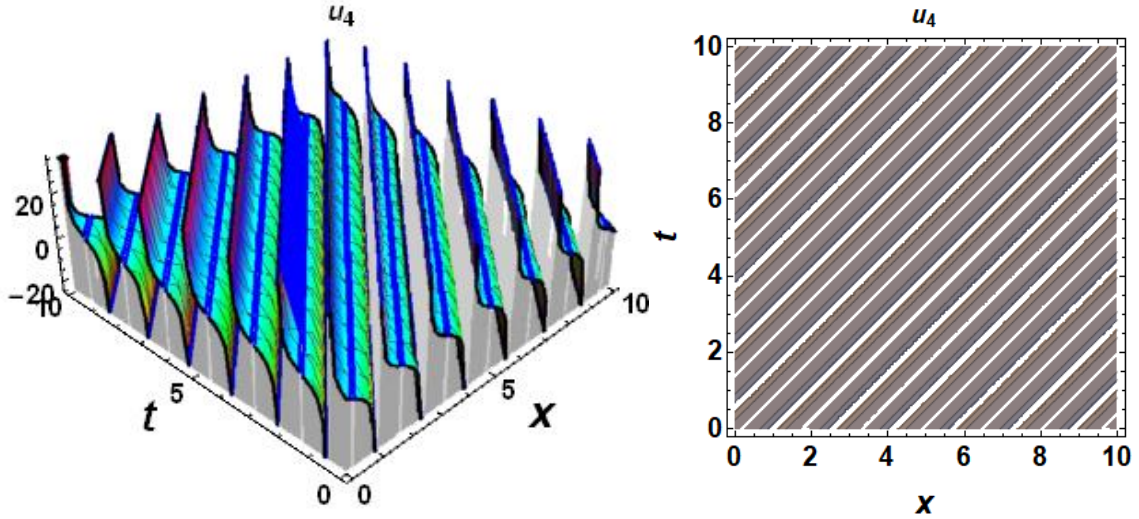


Figure 4. Graphs of $u_4(x,y,z,t) = u_4$ for $\lambda = 16,\ c = 1,\ \ a_0 = 5,\ \gamma = 1,\ \alpha = 1,\ \mu = 1,\ c_1 = -1,\ c_2 = 1,\ \mathrm{y} = 0, \mathrm{z} = 0.$

Case 5.

$$a_1 = \alpha,\ b_1 = \alpha\sqrt{\lambda}\sqrt{\sigma}, c = \frac{-3\alpha\gamma - \alpha^3\beta\lambda}{2\beta}, \mu = 0,\ \xi = \alpha x + \beta y + \gamma z - ct, \tag{24}$$

replacing values Eq. (24) into Eq. (14) and attain the following trigonometric exact solution for Eq. (1):

$$u_5(x,y,z,t)=\frac{\sqrt{{c_1}^2+{c_2}^2}\alpha\sqrt{\lambda}}{c_1\mathrm{Cos}[\sqrt{\lambda}\xi]+c_2\mathrm{Sin}[\sqrt{\lambda}\xi]}+\frac{\alpha(c_2\sqrt{\lambda}\mathrm{Cos}[\sqrt{\lambda}\xi]-c_1\sqrt{\lambda}\mathrm{Sin}[\sqrt{\lambda}\xi])}{c_1\mathrm{Cos}[\sqrt{\lambda}\xi]+c_2\mathrm{Sin}[\sqrt{\lambda}\xi]}+a_0. \tag{25}$$

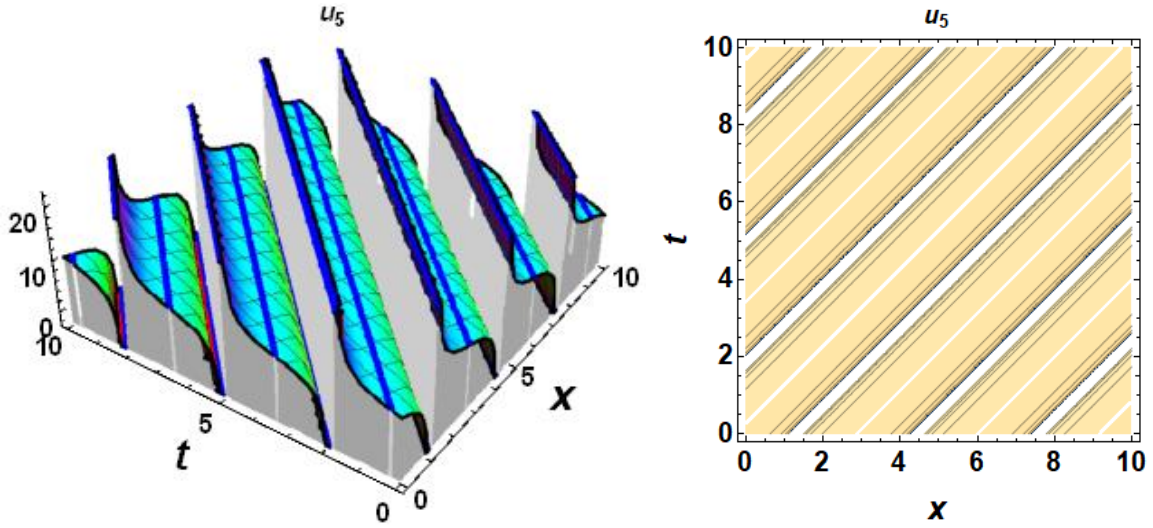


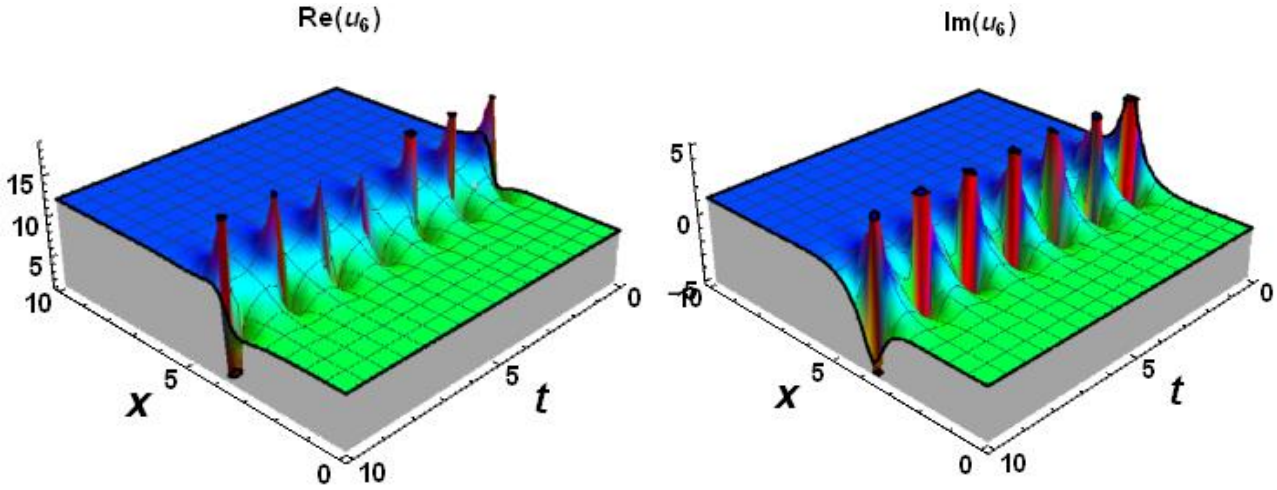
Figure 5. Graphs of $u_5(x,y,z,t)=u_5$ for $\lambda=4,\ \beta=10,\ \gamma=1,\ \alpha=1,\ a_0=10,\ c_1=1, c_2=-2,\ \mathrm{y}=1,\ \mathrm{z}=1,$ $c=1$.

Case 6.

$$a_1=\alpha,\ b_1=\alpha\sqrt{\lambda}\sqrt{\sigma},\ \mu=0,\ \beta=-\frac{3\alpha\gamma}{2c+\alpha^3\lambda}, \xi=\alpha x+\beta y+\gamma z-ct, \tag{26}$$

replacing values Eq. (26) into Eq. (14) and attain the following complex trigonometric exact solution for Eq. (1):

$$u_6(x,y,z,t)=\frac{\sqrt{c_1^2+c_2^2}\alpha\sqrt{\lambda}}{c_1\mathrm{Cos}[\sqrt{\lambda}(-ct+x\alpha+z\gamma-\frac{3y\alpha\gamma}{2c+\alpha^3\lambda})]+c_2\mathrm{Sin}[\sqrt{\lambda}(-ct+x\alpha+z\gamma-\frac{3y\alpha\gamma}{2c+\alpha^3\lambda})]}+$$
$$\frac{\alpha(c_2\sqrt{\lambda}\mathrm{Cos}[\sqrt{\lambda}(-ct+x\alpha+z\gamma-\frac{3y\alpha\gamma}{2c+\alpha^3\lambda})]-c_1\sqrt{\lambda}\mathrm{Sin}[\sqrt{\lambda}(-ct+x\alpha+z\gamma-\frac{3y\alpha\gamma}{2c+\alpha^3\lambda})])}{c_1\mathrm{Cos}[\sqrt{\lambda}(-ct+x\alpha+z\gamma-\frac{3y\alpha\gamma}{2c+\alpha^3\lambda})]+c_2\mathrm{Sin}[\sqrt{\lambda}(-ct+x\alpha+z\gamma-\frac{3y\alpha\gamma}{2c+\alpha^3\lambda})]}+a_0. \tag{27}$$



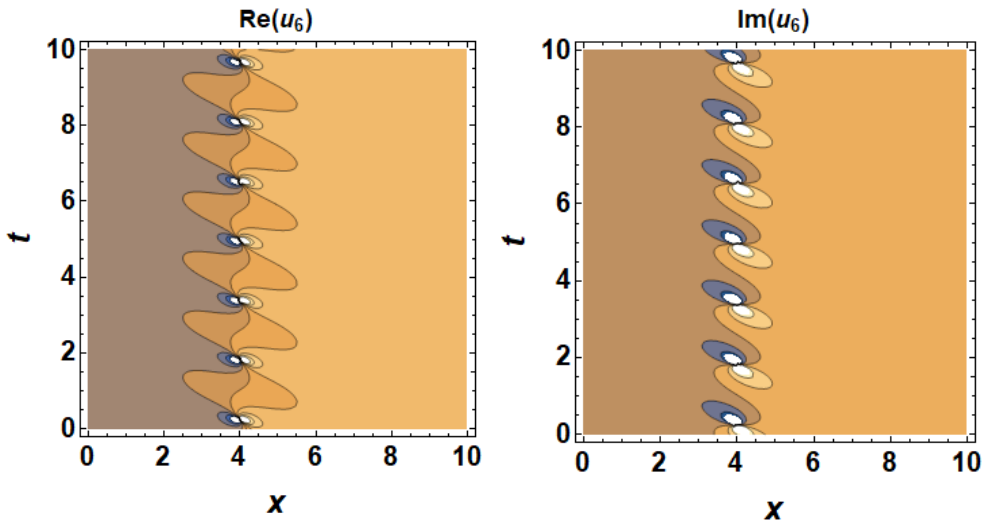


Figure 6. Real and imaginary graphs of $u_6(x,y,z,t) = u_6$ for $\lambda = 4,\ c = 1,\ a_0 = 10,\ \gamma = 8,\ \ c_1 = 2,\ c_2 = 10,\ \mathrm{y} = 1,\ \mathrm{z} = 1$.

iii) If $\lambda = 0$,

Case 7.

$$a_1 = \alpha,\ b_1 = -\sqrt{{c_2}^2\alpha^2 - 2\,c_1\alpha^2\mu},\ c = -\frac{3\alpha\gamma}{2\beta},\ \xi = \alpha x + \beta y + \gamma z - ct, \tag{28}$$

replacing values Eq. (28) into Eq. (14) and attain the following rational exact solution for Eq. (1):

$$u_7(x,y,z,t) = -\frac{\sqrt{{c_2}^2\alpha^2 - 2\,c_1\alpha^2\mu}}{c_1 + c_2\left(x\alpha+y\beta+z\gamma+\frac{3t\alpha\gamma}{2\beta}\right)+\frac{1}{2}\left(x\alpha+y\beta+z\gamma+\frac{3t\alpha\gamma}{2\beta}\right)^2\mu} + \frac{\alpha\left(c_2+\left(x\alpha+y\beta+z\gamma+\frac{3t\alpha\gamma}{2\beta}\right)\mu\right)}{c_1 + c_2\left(x\alpha+y\beta+z\gamma+\frac{3t\alpha\gamma}{2\beta}\right)+\frac{1}{2}\left(x\alpha+y\beta+z\gamma+\frac{3t\alpha\gamma}{2\beta}\right)^2\mu} + a_0. \tag{29}$$

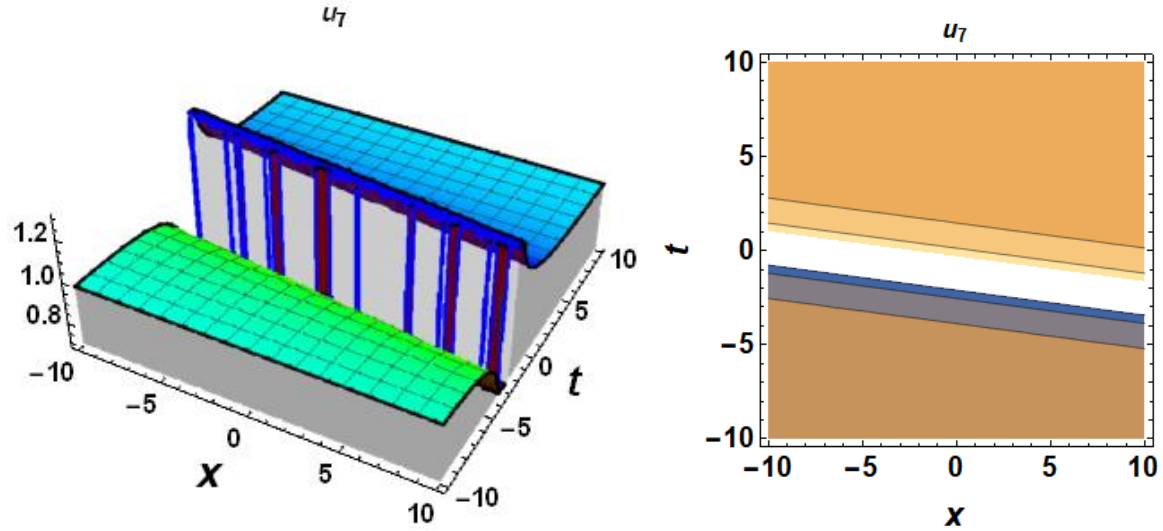


Figure 7. Graphs of $u_7(x,y,z,t) = u_7$ for $\mu = 2,\ \beta = 1,\ a_0 = 10,\ \gamma = 5,\ \ \alpha = 2,\ \ c_1 = 3,\ \ c_2 = 2,\ \mathrm{y} = 1,\ \mathrm{z} = 1$.

Case 8.

$$a_1 = 2\alpha,\ b_1 = 0,\ c = -\frac{3\alpha\gamma}{2\beta},\ \mu = 0,\ \xi = \alpha x + \beta y + \gamma z - ct\,, \tag{30}$$

replacing values Eq. (30) into Eq. (14) and attain the following rational exact solution for Eq. (1):

$$u_8(x,y,z,t) = \frac{2\,c_2\alpha}{c_1 + c_2\left(x\alpha + y\beta + z\gamma + \frac{3t\alpha\gamma}{2\beta}\right)} + a_0. \tag{31}$$

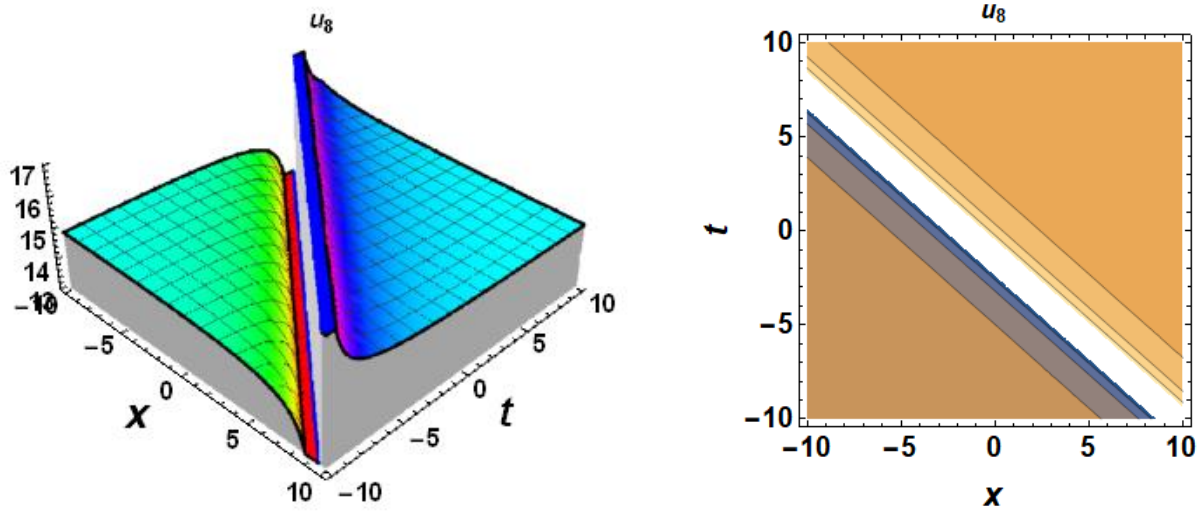


Figure 8. Graphs of $u_8(x,y,z,t) = u_8$ for $\beta = 4,\ a_0 = 15,\ \gamma = 3,\ a_0 = 5,\ \alpha = 5,\ c_1 = 1,\ c_2 = 1,\ \mathrm{y} = 1,\ \mathrm{z} = 1.$

Case 9.

$$a_1 = 2\alpha,\ b_1 = 0,\ \mu = 0,\ \beta = -\frac{3\alpha\gamma}{2c},\ \xi = \alpha x + \beta y + \gamma z - ct, \tag{32}$$

replacing values Eq. (32) into Eq. (14) and attain the following rational exact solution for Eq. (1):

$$u_9(x,y,z,t) = \frac{2\,c_2\alpha}{c_1 + c_2\left(-ct + x\alpha + z\gamma - \frac{3y\alpha\gamma}{2c}\right)} + a_0. \tag{33}$$

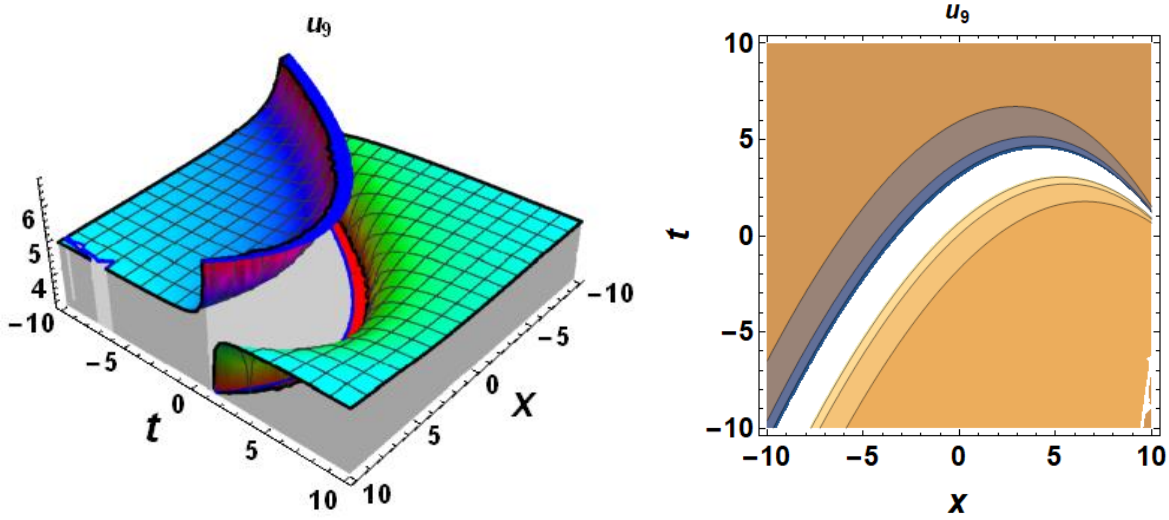


Figure 9. Graphs of $u_9(x,y,z,t) = u_9$ for $c = 1,\ \beta = 1,\ a_0 = 5,\ \gamma = 4,\ \alpha = 7,\ c_1 = 5,\ c_2 = 3,\ \mathrm{z} = 1.5,\ \mathrm{y} = 2.$

4. Conclusion

The $(G'/G, 1/G)$-expansion method was used to establish hyperbolic, trigonometric, complex trigonometric, and rational traveling wave solutions for the (3+1DJM) equation. For the solutions found, real and imaginary graphics are presented for different values given to the constants. This equation has been presented by many authors by applying different methods, traveling wave solutions. The $(G'/G, 1/G)$-expansion method was applied to the (3+1DJM) equation for the first time and the solutions produced by this method are of different types. The traveling wave solutions produced by the $(G'/G, 1/G)$-expansion method are trigonometric, complex trigonometric, hyperbolic and rational type solutions. There is a singular point within these solutions. Therefore, constants in solutions presented in shock wave theory and asymptotic behavior analysis will be much more valuable when the physical meaning is loaded. In this study, the applied method is effective, powerful, and will be used in future works to establish new exact solutions of many other NLEEs. Also, the ready computer package program is used for graphics and computations in this letter.

Acknowledgment

We would like to thank Fırat University and Ardahan University for their support to our study.

Conflict of Interest Statement

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Author Contribution Statement: All authors have contributed equally to the manuscript.

References

[1] Ekici M., Mirzazadeh M., Sonmezoglu A., Ullah M. Z., Zhou Q., Triki H., Biswas A., Optical solitons with anti-cubic nonlinearity by extended trial equation method, *Optik*, 136 (2017) 368-373.

[2] Rezazadeh H., Tariq H., Eslami M., Mirzazadeh M., Zhou Q., New exact solutions of nonlinear conformable time-fractional Phi-4 equation, *Chinese Journal of Physics*, 56 (6) (2018) 2805-2816.

[3] Durur H., Different types analytic solutions of the (1+1)-dimensional resonant nonlinear Schrödinger's equation using (G′/G)-expansion method, *Modern Physics Letters B*, *34*(03) (2020) 2050036.

[4] Yokus A., Kuzu B., Demiroğlu U., Investigation of solitary wave solutions for the (3+1)-dimensional Zakharov - Kuznetsov equation. *International Journal of Modern Physics B*, 33(29) (2019) 1950350.

[5] Su-Ping Q., Li-Xin T., Modification of the Clarkson–Kruskal Direct Method for a Coupled System, *Chinese Physics Letters*, 24 (10) (2007) 2720.

[6] Manafian J., Lakestani, M., Abundant soliton solutions for the Kundu–Eckhaus equation via tan (ϕ (ξ))-expansion method, *Optik*, 127 (14) (2016) 5543-5551.

[7] Yavuz M., Özdemir N., An Integral Transform Solution for Fractional Advection-Diffusion Problem, *Mathematical Studies and Applications*, 442 (2018) 4-6 October.

[8] Duran S., Kaya D., Applications of a new expansion method for finding wave solutions of nonlinear differential equations, *World Applied Sciences Journal*, 18 (11) (2012) 1582-1592.

[9] Cattani C., Sulaiman T. A., Baskonus H. M., Bulut H., On the soliton solutions to the Nizhnik-Novikov-Veselov and the Drinfel'd-Sokolov systems, *Optical and Quantum Electronics*, 50 (3) (2018) 138.

[10] Kumar D., Seadawy A. R., Joardar A. K., Modified Kudryashov method via new exact solutions for some conformable fractional differential equations arising in mathematical biology, *Chinese journal of physics*, 56 (1) (2018) 75-85.

[11] Yokuş A., Durur H., Complex hyperbolic traveling wave solutions of Kuramoto-Sivashinsky equation using (1/G') expansion method for nonlinear dynamic theory, *Journal of Balıkesir University Institute of Science and Technology*, 21 (2) (2019) 590-599.

[12] Yokus A., Durur H., Ahmad H., Thounthong, P., Zhang Y. F., Construction of exact traveling wave solutions of the Bogoyavlenskii equation by (G′/G,1/G)-expansion and (1/G′)-expansion techniques, *Results in Physics*, 19 (2020) 103409.

[13] Yokus A., Durur H., Ahmad H., Hyperbolic type solutions for the couple Boiti-Leon-Pempinelli system, *Facta Universitatis, Series: Mathematics and Informatics*, 35 (2) (2020) 523-531.

[14] Durur H., Yokuş A., Analytical solutions of Kolmogorov–Petrovskii–Piskunov equation, *Journal of Balikesir University Institute of Science and Technology*, 22 (2) (2020) 628-636.

[15] Yokus A., Durur H., Ahmad H., Yao S. W., Construction of Different Types Analytic Solutions for the Zhiber-Shabat Equation, *Mathematics*, 8 (6) (2020) 908.

[16] Aziz I., Šarler, B., The numerical solution of second-order boundary-value problems by collocation method with the Haar wavelets, *Mathematical and Computer Modelling*, 52 (9-10) (2010) 1577-1590.

[17] Aziz I., Asif M., Haar wavelet collocation method for three-dimensional elliptic partial differential equations, *Computers & Mathematics with Applications*, 73 (9) (2017) 2023-2034.

[18] Darvishi M., Arbabi S., Najafi M., Wazwaz, A., Traveling wave solutions of a (2+1)-dimensional Zakharov-like equation by the first integral method and the tanh method, *Optik*, 127 (16) (2016) 6312-6321.

[19] Gao W., Silambarasan R., Baskonus H. M., Anand R. V. and Rezazadeh H., Periodic waves of the non-dissipative double dispersive micro strain wave in the micro structured solids, *Physica A: Statistical Mechanics and its Applications*, 545 (2020).

[20] Duran S., Askin M., Sulaiman T. A., New soliton properties to the ill-posed Boussinesq equation arising in nonlinear physical science, *An International Journal of Optimization and Control: Theories & Applications (IJOCTA)*, 7 (3) (2017) 240-247.

[21] Faraj B., Modanli M., Using Difference Scheme Method for the Numerical Solution of Telegraph Partial Differential Equation, (2017).

[22] Çelik N., Seadawy A. R., Özkan Y. S., Yaşar E., A model of solitary waves in a nonlinear elastic circular rod: Abundant different type exact solutions and conservation laws, *Chaos, Solitons & Fractals*, 143 (2021) 110486.

[23] Duran S., Solitary Wave Solutions of the Coupled Konno-Oono Equation by using the Functional Variable Method and the Two Variables (G'/G, 1/G)-Expansion Method, *Adıyaman University Journal of Science*, 10 (2) (2020) 585-594.

[24] Yavuz M., Sulaiman T. A., Yusuf A., Abdeljawad T., The Schrödinger-KdV equation of fractional order with Mittag-Leffler nonsingular kernel. *Alexandria Engineering Journal*, 60(2) (2021) 2715-2724.

[25] Evirgen F., Uçar S., Özdemir N., Hammouch Z., System response of an alcoholism model under the effect of immigration via non-singular kernel derivative, *Discrete and Continuous Dynamical Systems-S*, (2018) 10.

[26] Kaya D., Bulut H., Inc, M., Numerical Study of a Klein-Gordon Equation by The Adomian's Decompositon Method, *Hadronic Journal*, 28 (3) (2005) 311.

[27] Duran S., Exact Solutions for Time-Fractional Ramani and Jimbo-Miwa Equations by Direct Algebraic Method, *Advanced Science, Engineering and Medicine*, 12 (7) (2020) 982-988.

[28] Yokuş A., Durur H., Abro K. A., Kaya D., Role of Gilson–Pickering equation for the different types of soliton solutions: a nonlinear analysis, *The European Physical Journal Plus*, 135 (8) (2020) 1-19.

[29] Duran S., Kaya D., New Wave Solutions for Nonlinear Differential Equations using an Extended Bernoulli Equation as a New Expansion Method, In ITM Web of Conferences, Kıbrıs-Girne, (2018), (22) 01035 1-5.

[30] Siddique I., Rizvi, S. T. R., Batool, F., New exact travelling wave solutions of nonlinear evolution equations, *International Journal of Nonlinear Science*, 9 (1) (2010) 12-18.

[31] Jimbo M., Miwa T., Solitons and infinite dimensional Lie algebras, Publications of the Research Institute for Mathematical Sciences, 19 (3) (1983) 943-1001.

[32] Wazwaz A. M., Multiple-soliton solutions for extended (3+1)-dimensional Jimbo–Miwa equations, *Applied Mathematics Letters*, 64 (2017) 21-26.

[33] Zhang S., New periodic wave solutions of a (3+1)-dimensional Jimbo–Miwa equation. *SN Applied Sciences*, 1 (3) (2019) 201.

[34] Ma W. X., Lee J. H., A transformed rational function method and exact solutions to the 3+1 dimensional Jimbo–Miwa equation, *Chaos, Solitons & Fractals*, 42 (3) (2009) 1356-1363.

[35] Sun H. Q., Chen, A. H., Lump and lump–kink solutions of the (3+1)-dimensional Jimbo–Miwa and two extended Jimbo–Miwa equations, *Applied Mathematics Letters*, 68 (2017) 55-61.

[36] Yang J. Y., Ma W. X., Abundant lump-type solutions of the Jimbo–Miwa equation in (3+1)-dimensions, *Computers & Mathematics with Applications*, 73 (2) (2017) 220-225.

[37] Liu X. Q., Jiang S., New solutions of the 3+1 dimensional Jimbo–Miwa equation. *Applied mathematics and computation*, 158 (1) (2004) 177-184.

[38] Öziş T., Aslan İ., Exact and explicit solutions to the (3+ 1)-dimensional Jimbo–Miwa equation via the Exp-function method, *Physics Letters A,* 372 (47) (2008) 7011-7015.

[39] Tang X. Y., Liang Z. F., Variable separation solutions for the (3+1)-dimensional Jimbo–Miwa equation, *Physics Letters A,* 351 (6) (2006) 398-402.

[40] Ma W. X., Lump-type solutions to the (3+1)-dimensional Jimbo-Miwa equation, *International Journal of Nonlinear Sciences and Numerical Simulation*, 17 (7-8) (2016) 355-359.